MENIIT
NEET | IIT-JEE | FOUNDATION
Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 20-07-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

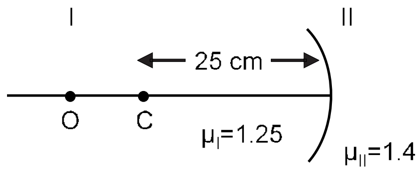
QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Region I and II are separated by a spherical surface of radius 25 cm. an object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface.



(1) 9.52 cm (2) 18.23 cm (3) 55.44 cm (4) 37.58 cm

Ans. (4)

Sol. $\frac{n_2}{v} \quad \frac{n_1}{u} \quad \frac{n_2 \quad n_1}{R}$

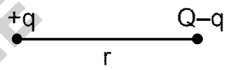
$\frac{1.4}{v} \quad \frac{1.25}{40} \quad \frac{1.4 \quad 1.25}{(\ 25)} \quad v \quad 37.58\text{cm}$

2. A certain charge Q is divided into two parts q and (Q – q). How should the charges Q and q be divided so that q and (Q – q). Placed at a certain distance apart experience maximum electrostatics repulsion.

(1) Q = 4q (2) Q = 3q (3) Q $\frac{q}{2}$ (4) Q = 2q

Ans. (4)

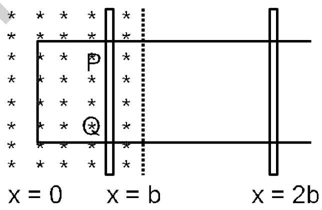
Sol. F $\frac{Kq_1q_2}{r^2} \quad \frac{K(q)(Q \quad q)}{r^2}$

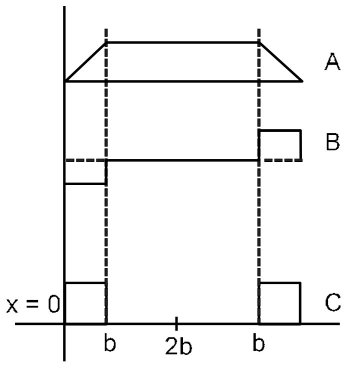


$\frac{dF}{dq} \quad 0$

Q – 2q = 0 Q = 2q

3. The arm PQ of a rectangular conductor is moving from x = 0 to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph showing the variation of different quantities with distance.





(1) A-Flux, B-Power dissipated, C-EMF (2) A-EMF, B-Power dissipated, C-Flux

(3) A-Power dissipated, B-Flux, C-EMF (4) A-Flux, B-EMF, C-Power dissipated

Ans. (4)

Sol. Flux = ϕ = B.A

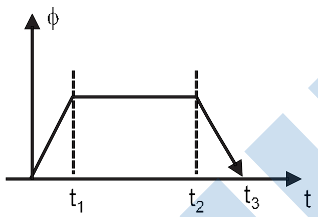
$\Rightarrow$ B × A cos0

Where A = ℓvt

$\phi = B\ell vt$

One rod go at x > b then ϕ stop changing this constant flux = $B\ell b$.

When rod come back and when x < b flux start decreasing so graph ϕv/st

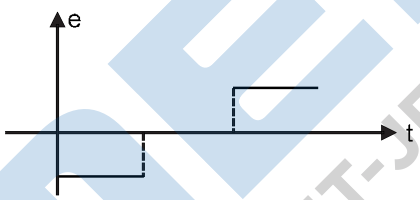


b $\rightarrow$ (ii)

$e \quad \frac{d}{dt}$; e = – slope of ϕ – t graph

In 0 – t_1 graph slope +ve and constant so e = negative and zero.

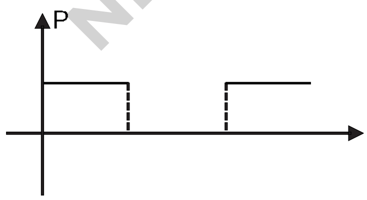
in $t_2 - t_3$ slope of ϕ – t is negative and constant so e = positive and zero



Power = e^2/R

Resistance is only of rod so R of the circuit is constant

$P \quad \frac{B^2\ell^2v^2}{R} \quad$ constant



4. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A} \quad \vec{B} \quad |\vec{A} \quad \vec{B}|$. Then the value of $|\vec{A} \quad \vec{B}|$ will be:

(1) $\sqrt{A^2 \quad B^2 \quad 2AB}$ (2) $\sqrt{A^2 \quad B^2 \quad \sqrt{2}AB}$ (3) $\sqrt{A^2 \quad B^2 \quad \sqrt{2}AB}$ (4) $\sqrt{A^2 \quad B^2 \quad \sqrt{2}AB}$

Ans. (2)

Sol. $\vec{A}\ \vec{B}\ \ |\vec{A}\ \ \vec{B}|$

$\Rightarrow$ AB cos θ = AB sin θ

$\therefore$ θ = 45°

$\therefore$ $|\vec{A}\ \ \vec{B}|\ \ \sqrt{A^2\ \ B^2\ \ 2AB\cos 45^\circ}$

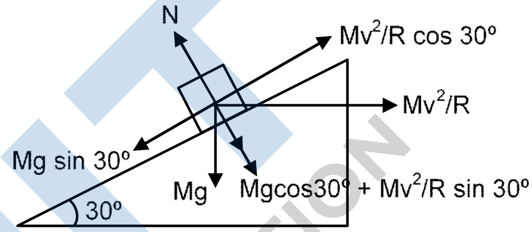
$\sqrt{A^2\ \ B^2\ \ \sqrt{2}AB}$

5. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ × 10^3 kg m/s^2. (Given cos 30° = 0.87, μ_s = 0.2)

(1) 7.2 (2) 12.4 (3) 10.2 (4) 6.96

Ans. (3)

Sol. Perpendicular to inclined plane



$$N\ \ mg\cos 30^\circ\ \ \frac{mv^2}{R}\sin 30^\circ$$

$$N\ \ mg\cos 30^\circ\ \ \frac{mv^2}{R}\sin 30^\circ \quad ....(1)$$

along inclined plane

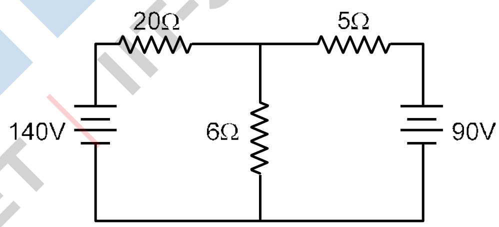
$$mg\sin 30^\circ\ \ \mu_s N\ \ \frac{mv^2}{R}\cos 30^\circ \quad ....(2)$$

Dividing (1) by (2)

$$\frac{N\ \ mg\cos 30^\circ}{mg\sin 30^\circ\ \ \mu_s N}\ \ \tan 30^\circ$$

N = 10.2 × 10^3 kg / ms^2

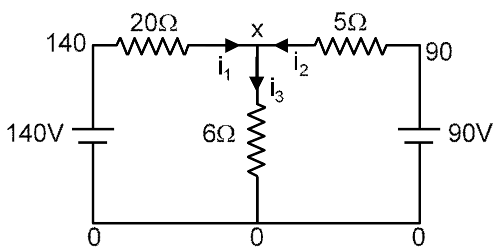
6. The value of current in the 6Ω resistance is :



(1) 4 A (2) 10 A (3) 8 A (4) 6 A

Ans. (2)

Sol. Let potential at junction point = x



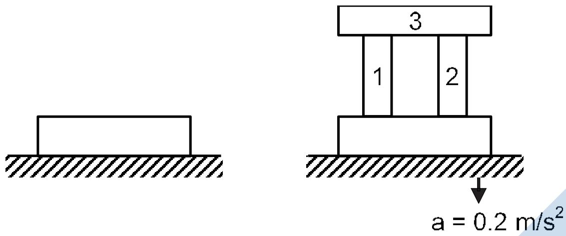
By KCL $\quad i_{in}\ \ 0\ \ \frac{140\ \ x}{20}\ \ \frac{90\ \ x}{5}\ \ \frac{0\ \ x}{6}\ \ 0$

x = 60 V

So current $i_3 \quad \frac{x}{6} \quad \frac{60}{6} \quad 10A$

7. A steel block of 10 kg rests on horizontal floor as shown. When three iron cylinders are placed on its as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. Then normal reaction R' by the floor if mass of the iron cylinders are equal and of 20 kg each, is _______ N.

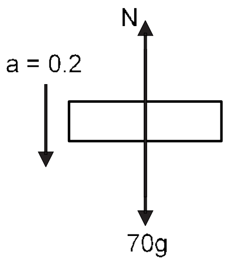
(Take g = 10 m/s^2 and μ_s = 0.2)



(1) 686 (2) 714 (3) 684 (4) 716

Ans. (1)

Sol.



70g – N = 70 × 0.2

N = 700 – 14 ; N = 686

8. The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0ºC to 50ºC when no work is done is ______. (R is the universal gas constant)

(1) 750 R (2) 500 R (3) 250 R (4) 175 R

Ans. (2)

Sol. n = 4

ΔT = 50K

As W – 0. It means isochoric process

Q = ΔU

$nC_v \quad T \quad 4 \quad \frac{5R}{2} \quad 50 \quad 500R$

9. The radiation corresponding to 3 $\rightarrow$ 2 transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5 × 10^{-4} T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is:

(Mass of electron = 9.1 × 10^{-31} kg)

(1) 0.82 eV (2) 0.16 eV (3) 1.36 eV (4) 1.88 eV

Ans. (1)

Sol. $E_p = 13.6\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) eV$

$= 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$

Ep = 1.89 eV

For gold plate

$\phi = Ep - KEmax$

$v = \frac{RqB}{m}$

$= \frac{7\times10^{-3}\times1.6\times10^{-19}\times5\times10^{-4}}{9.1\times10^{-31}} = 6.15\times10^{5}$

$KE = \frac{1}{2}mV^2$

$KE = \frac{1}{2}\frac{9.1\times10^{-31}\times(6.15\times10^{5})^2}{1.6\times10^{-19}} eV = 1.075 eV$

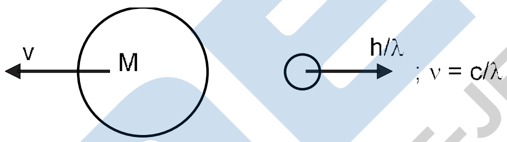
$\phi = 1.89 - 1.075$; $\phi = 0.82$ eV

10. A nucleus of mass M emits γ-rays photon of frequency 'ν'. The loss of internal energy by the nucleus is: [Take 'c' as the speed of electromagnetic wave]

(1) 0 (2) $h\nu\left(1 + \frac{h\nu}{2Mc^2}\right)$ (3) $h\nu$ (4) $h\nu\left(1 - \frac{h\nu}{2Mc^2}\right)$

Ans. (2)

Sol.



$Mv = \frac{h}{\lambda} = \frac{h\nu}{c}$

Loss of energy $= \frac{1}{2}Mv^2 + h\nu = \frac{1}{2}\frac{p^2}{M} + h\nu = \frac{1}{2M}\left(\frac{h\nu}{c}\right)^2 + h\nu = h\nu\left(1 + \frac{h\nu}{2Mc^2}\right)$

11. Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

(1) $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$ (2) $v_A = v_B = v_C = 0$ (3) $v_A = v_B \neq v_C$ (4) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

Ans. (4)

Sol. $V_{rms} = \sqrt{\frac{3RT}{m}}$

12. AC voltage V(t) = 20 sinωt of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2. The amplitude of the oscillating displacement current for the applied AC voltage is ________. (Take ε_0 = 8.85 × 10^{-12} F/m)

(1) 55.58 μ_A (2) 21.14 μ_A (3) 27.79 μA (4) 83.37 μA

Ans. (3)

Sol. $I_{dis} = \epsilon_0 \frac{d\phi_E}{dt}$

$$\phi_E = \frac{V(t)A}{d} = \frac{20\sin 100\pi t.1}{2\times 10^{-3}} = 10^4 \sin 100\pi t$$

$$I_{dis} = \epsilon_0 \frac{d}{dt}(10^4 \sin 100\pi t) = 8.85 \times 10^{-12} \times 10^4 \times 100\pi \cos 100\pi t = 27.79\ \mu A \cos 100\pi t$$

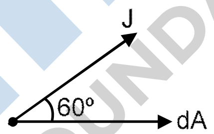
13. A current of 5A is a passing through a non-linear magnesium wire of cross-section 0.04 m^2. At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is :

(Resistivity of magnesium ρ = 44 × 10^{-8} Ωm)

(1) 11 × 10^{-7} V/m (2) 11 × 10^{-5} V/m (3) 11 × 10^{-2} V/m (4) 11 × 10^{-3} V/m

Ans. (2)

Sol. $di = JdA\cos\theta = \frac{JdA}{2}; i = \frac{EA}{2\rho}$



$$E = \frac{2\rho i}{A} = \frac{2\times 44\times 10^{-8}\times 5}{4\times 10^{-2}} = 11\times 10^{-5}\ V/m$$

14. The entropy of any system is given by $S = \alpha^2\beta \ln\left[\frac{\mu kR}{J\beta^2} + 3\right]$, where α and β are the constants. μ, J. k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively. [Take $S = \frac{dQ}{T}$] Choose the incorrect options from the following :

(1) S and α have different dimensions. (2) S, β, k and μR have the same dimensions.

(3) α and J have the same dimensions. (4) α and k have the same dimensions.

Ans. (4)

Sol. $S = \frac{Q}{T}$

$$[S] = \frac{ML^2T^{-2}}{K}$$

$$K = \frac{Energy}{T}$$

$$[K] = [S] = \frac{ML^2T^{-2}}{K} \qquad [R] = \frac{Energy}{nT} = \frac{ML^2T^{-2}}{molK}$$

$[J] = M^0L^0T^0$

Now, $[\mu KR] = [J\beta^2]$; (mol) $\frac{ML^2T^{\ 2}}{K}$ $\frac{ML^2T^{\ 2}}{molK}$ $[\ ^2]$

$[\beta] = ML^2T^{-2}K^{-1}$; $[\ ^2]$ $\frac{S}{\ }$ $\frac{ML^2T^{\ 2}}{K\ \ ML^2T^{\ 2}K^{\ 1}}$; $M^0L^0T^0$

15. A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic filed. Let r_d and r_a be their respective radii of circular path. The value of $\frac{r_d}{r_a}$ is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) 2 (4) 1

Ans. (2)

Sol. $r \quad \frac{mv}{qB} \quad \frac{\sqrt{2mK}}{qB}$

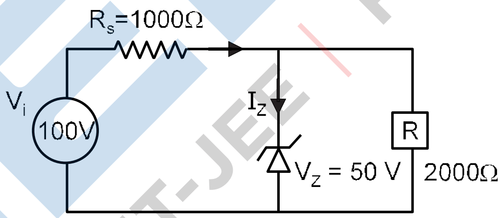
$r \quad \frac{\sqrt{m}}{q}$

$m_{\propto} = 2m_d$

$_{d\propto} = 2d_d$

$\frac{r_d}{r} \quad \frac{\sqrt{m_d}}{q_d} \quad \frac{2q_d}{\sqrt{2m_d}} \quad \sqrt{2}$

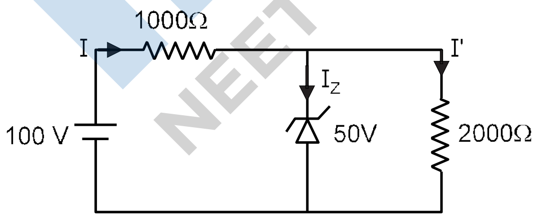
16. For the circuit shown below, calculate the value of I_z :



(1) 0.15 A (2) 25 mA (3) 0.05 A (4) 0.1 A

Ans. (2)

Sol.



$I \quad \frac{100 \quad 50}{1000} \quad 50mA$

$I' \quad \frac{50}{2000} \quad 0.025mA$

$I = I_z + I'$

$I_z = I - I' = 50 \text{ mA} - 25 \text{ mA} = 25 \text{ mA}$

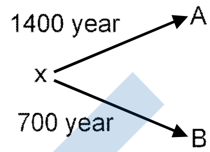
17. A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains ?

(Take ln 3 = 1.1)

(1) 700 years (2) 340 years (3) 740 years (4) 1110 years

Ans. (3)

Sol. $\frac{dN_x}{dt} \quad {}_1N_x \quad {}_2N_x$



$\int_N^{N/3} \frac{dN_x}{N_x} \quad \int_0^1 (\ _1 \quad _2)dt$

$\ln 3 \quad \frac{\ln 2}{1400} \quad \frac{\ln 2}{700} \quad t$

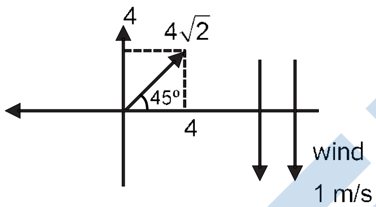
$t \quad \frac{\ln 3}{\ln 2} \quad \frac{1400}{3} \quad 740\text{year}$

18. A butterfly is flying with a velocity 2 4 m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is :

(1) $12\sqrt{2}$ m (2) 15 m (3) 3 m (4) 20 m

Ans. (2)

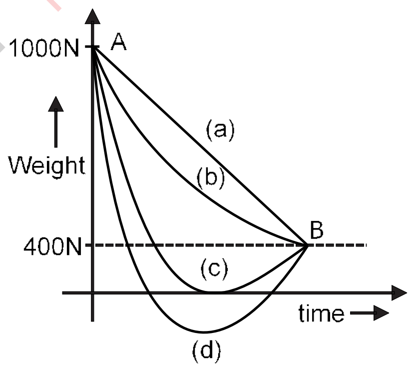
Sol.



$\vec{D} \quad v_{EG} \quad T \qquad 4\hat{i} \quad 4\hat{j} \quad (\ \hat{j}) \quad 3s \quad (4\hat{i} \quad 3\hat{j}) \quad 3s$

$|\vec{D}| \quad 15m$

19. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.



(1) (c) (2) (d) (3) (b) (4) (a)

Ans. (1)

Sol. $\vec{g}$ (at any point) $\vec{g}_{Earth} \quad \vec{g}_{mars}$. Since distance is large so $|\vec{g}| \quad |\vec{g}_E| \quad 0$.

As we move away from earth, it decrease to zero at a point where $\vec{g}_E \quad \vec{g}_M \quad 0$

Then it increase to $|\vec{g}| \quad |\vec{g}_M| \quad 4$ at mars surface.

20. The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2 respectively. The actual length of the metal wire is :

(1) $\frac{T_1 l_1 \quad T_2 l_2}{T_1 \quad T_2}$ (2) $\frac{l_1 \quad l_2}{2}$ (3) $\frac{T_1 l_2 \quad T_2 l_1}{T_1 \quad T_2}$ (4) $\sqrt{T_1 T_2 l_1 l_2}$

Ans. (3)

Sol. Let initial length of rod be L_0 and Area A.

As $\frac{T}{A} \quad Y\frac{\ell}{\ell}$

So, $\frac{T_1}{A} \quad \frac{Y(L_1 \quad L_0)}{L_0}$

$\frac{T_2}{A} \quad \frac{Y(L_2 \quad L_0)}{L_0}$

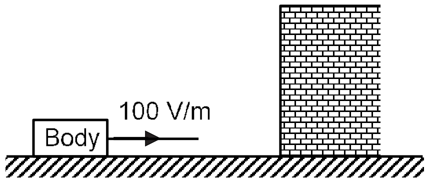
Dividing

$\frac{T_1}{T_2} \quad \frac{L_1 \quad L_0}{L_2 \quad L_0}; T_1L_2 \quad T_1L_0 \quad T_2L_1 \quad T_2L_0; \quad \frac{L_2T_1 \quad L_1T_2}{T_1 \quad T_2}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, the time period of the motion will be _______s.



Ans. (1)

Sol. $a \quad \frac{qE}{m} \quad \frac{8 \quad 10^{6}}{10^{3}} \quad 100 \quad 0.8m/s^2$

As electric field is switched on, ball first strikes to wall and returns back.

On oscillation.

Thus $s \quad ut_1 \quad \frac{1}{2}at_1^2$

$0.1 \quad \frac{1}{2} \quad 0.8t_1^2; t_1 \quad \frac{1}{2}s$

Thus time period $T \quad 2 \quad \frac{1}{2} \quad 1sec.$

2. An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to _______cm.

Ans. (25)

Sol. magnification of microscopic lens

$m \quad 1 \quad \frac{D}{f_0}; 6 \quad 1 \quad \frac{25}{f_0}; f_0 \quad 5cm$

Magnification of compound microscope when image formed at infinity

$m \quad \frac{\ell}{f_0} \quad \frac{D}{f_e}$

$12 \quad \frac{60}{5} \quad \frac{25}{f_0}; f_0 \quad 25cm$

3. A carrier wave $V_c(t) = 160 \sin(2\pi \times 10^6 t)$ volts is made to vary between $V_{max} = 200$ V and $V_{min} = 120$ by a message signal $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ volts. The peak voltage A_m of the modulating signal is _____

Ans. (40)

Sol. $V_{max} = A_C + A_m$

$\Rightarrow 200 = 160 + A_m$

$\Rightarrow A_m = 40$

4. The amplitude of wave disturbance propagating in the positive x-direction is given by $y \quad \frac{1}{(1 \quad x)^2}$ at time $t = 0$ and $y \quad \frac{1}{1 \quad (x \quad 2)^2}$ at t = 1 s, where x and y are in metres. The shape of wave does not change during the propagation. The velocity of the wave will be ____ m/s.

Ans. (2)

Sol. $x \rightarrow (x - vt)$

$$y \quad \frac{1}{1 \quad (x \quad vt)^2}$$

At $t \quad 0$; $y \quad \frac{1}{1 \quad x^2}$; at $t \quad 1sec$; $y \quad \frac{1}{1 \quad (x \quad v)^2}$

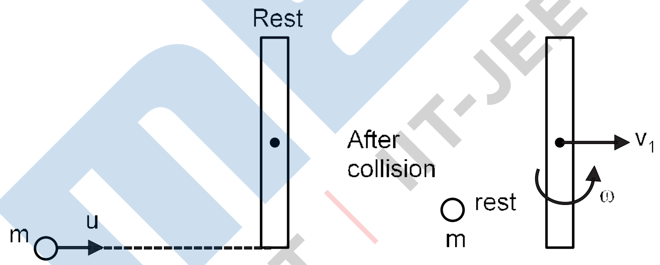
By comparing

V = 2 m/s

5. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio masses $\frac{m}{M}$ is $\frac{1}{x}$. The value of 'x' will be _______.

Ans. (4)

Sol.



Conservation of angular momentum about centre of mass of rod

$$mu \quad \frac{L}{2} \quad \frac{ML^2}{12}(\quad) \qquad \text{.... (1)}$$

$$mu = Mv_1 \qquad \text{.... (2)}$$

$$1 \quad \frac{v_1 \quad \frac{L}{2}}{u} \qquad \text{.... (3)}$$

Putting v_1 from (2) and ωL from (1) in (3)

$$u \quad \frac{m}{M}u \quad \frac{6mu}{2M}$$

$$1 \quad \frac{4m}{M}; m/M \quad 1/4$$

6. The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is _____ km/h.

Ans. (132)

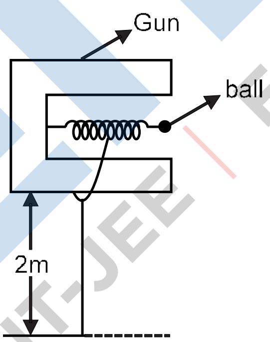
Sol. Frequency received by wall $f' \quad \frac{v_s}{v_s \quad v} \quad f_0$

Reflected frequency received by man is $f' \quad \frac{v_s \quad v}{v_s} \quad f'$

$$\Rightarrow \quad f'' \quad \frac{v_s \quad v}{v_s} \quad \frac{v_s}{v_s \quad v} \quad f_0 \quad f'' \quad \frac{v_s \quad v}{v_s \quad v} \quad f_0 \quad 500 \quad \frac{330 \quad v}{330 \quad v} \quad 400$$

$$\Rightarrow \quad v \quad \frac{330}{9} \quad \frac{18}{5} \quad 132 km/hr$$

7. In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is ______ m. ($g = 10\ m/s^2$)



Ans. (1)

Sol. By energy conservation

$$\frac{1}{2}kx^2 \quad \frac{1}{2}mv^2 \quad v \quad x\sqrt{\frac{k}{m}} \quad v \quad 0.05\sqrt{\frac{100}{0.1}} \quad 0.5\sqrt{10} m/s$$

Time of flight of ball $T \quad \sqrt{\frac{2H}{g}} \quad \sqrt{\frac{2 \quad 2}{10}} \quad \frac{2}{\sqrt{10}} sec$

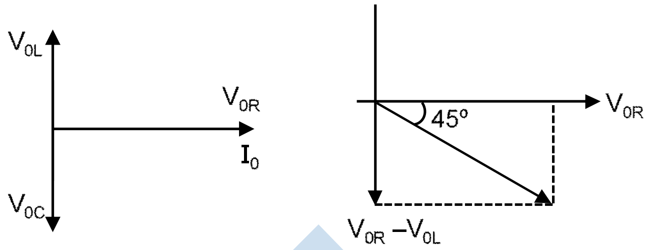
Range of ball s = ut

$$d \quad 0.5\sqrt{10} \quad \frac{2}{\sqrt{10}} \quad 1m$$

8. In an LCR series circuit, an inductor 30 mH and a resistor 1Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x} \quad 10^{\ 3}$F . Then the value of x is ______.

Ans. (3)

Sol. $\tan 45^\circ \quad \frac{V_{0C} \quad V_{0L}}{V_{0R}} \quad \frac{X_C \quad X_L}{R}$



$R \quad \frac{1}{C} \quad L$

$1 \quad \frac{1}{300C} \quad 30 \quad 10^{\ 3} \quad 300$

$C \quad \frac{1}{3} \quad 10^{\ 3}F$

9. A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it takes time 't_1'. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. Then value of x will be ________.

Ans. (2)

Sol. When disc slides a_1 gsinθ So S $\quad ut_1 \quad \frac{1}{2}a_1t_1^2 \quad \frac{1}{2}g\sin \ .t_1^2$ ....(1)

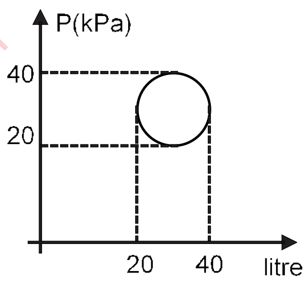
When disc do pure rolling $a_2 \quad \frac{g\sin}{1 \quad k^2/R^2} \quad \frac{g\sin}{1 \quad 1/2} \quad \frac{2}{3}g\sin$

So S $\quad ut_2 \quad \frac{1}{2}a_2t_2^2 \quad \frac{1}{2}.\frac{2}{3}g\sin \ .t_2^2$ ....(2)

From (1) & (2)

$\frac{t_2}{t_1} \quad \sqrt{\frac{3}{2}}$

10. In the reported figure, heat energy absorbed by a system in going through a cyclic process is _____ πJ.



Ans. (100)

Sol. ΔQ = W + ΔU = W = area enclosed by the curve
ΔQ = πab

$$\frac{40 \quad 20}{2} \quad 10^{3} \qquad \frac{40 \quad 20}{2} \quad 10^{\;3}$$

= 100 π Joule

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are :

(1) Primary amine, isonitrile compound, conc. HCl

(2) Secondary amine, nitrile compound, conc. NaOH

(3) Primary amine, nitrile compound, conc. HCl

(4) Secondary amine, isonitrile compound, conc. NaOH

Ans. (1)

Sol. Only 1° amines give carbylamines reaction

$$\underset{\substack{1^{\circ}\text{ amine} \\ (P)}}{R \quad NH_2} \quad CHCl_3 \quad 3KOH \qquad \underset{\substack{\text{Isonitrile} \\ \text{(Poisonous product)} \\ (Q)}}{R \quad NC} \quad 3KCl \quad 3H_2O$$

2. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Sharp glass becomes smooth on heating it upto its melting point.

Reason R : The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below :

(1) A is false but R is true.

(2) Both A and R are true but R is NOT the correct explanation of A.

(3) A is true but R is false.

(4) Both A and R are true and R is the correct explanation of A.

Ans. (2)

Sol. On heating viscosity decreases, but does not have any relation with smoothing of glass on heating.

3. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The dihedral angles in H_2O_2 in gaseous phase is 90.2° and in solid phase is 111.5°.

Reason R : The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.

Choose the most appropriate answer from the options given below for A and R.

(1) A is correct but R is not correct.

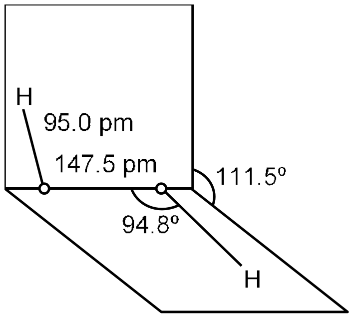
(2) Both A and R are correct and R is the correct explanation of A.

(3) A is not correct but R is correct.

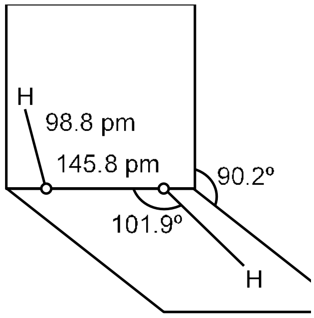
(4) Both A and R are correct but R is not the correct explanation of A.

Ans. (3)

Sol.



(a) Gas phase

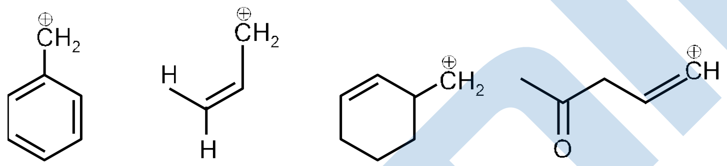


(b) Solid phase

(a) H_2O_2 structure in gas phase, dihedral angle is 111.5°. (b) H_2O_2 structure in solid phase at 110 K, dihedral angle is 90.2°.

The dihedral angle of H_2O_2 in gaseous phase is approximate 111.5º. While dihydral angle in solid H_2O_2 is affected by hydrogen bonding and it is 90.2º in solid state.

4.



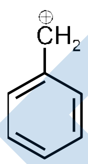
(A) (B) (C) (D)

Among the given species the Resonance stabilised carbocations are :

(1) (A), (B) and (C) only
(2) (A) and (B) only
(3) (C) and (D) only
(4) (A), (B) and (D) only

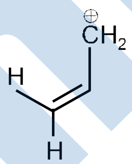
Ans. (2)

Sol.



(A)

&



(B)

are resonance stabilised carbocations.

5. Identify the incorrect statement from the following :

(1) Starch is a polymer of α-D glucose

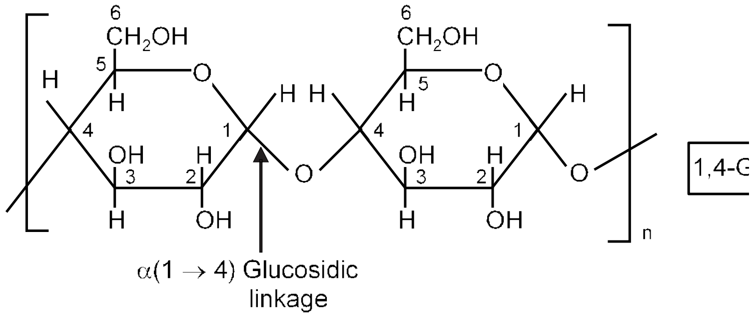
(2) Glycogen is called as animal starch

(3) β-Glycosidic linkage makes cellulose polymer

(4) Amylose is a branched chain polymer of glucose

Ans. (4)

Sol. The amylose molecule is made up of D-glucose unit joined by □-glycosidic linkages between C-1 of one glucose unit and C-4 of the next glucose unit.



Amylose

6. Green Chemistry in day-to-day life is in the use of :

(1) Liquified CO_2 for dry cleaning of clothes
(2) Large amount of water alone for washing clothes
(3) Tetrachloroethene for laundry
(4) Chlorine for bleaching of paper

Ans. (1)

Sol. $CCl_2 = CCl_2$ was earlier used as solvent for dry cleaning agent but it is carcinogen. So liquid CO_2 is used. Replacement of halogenated solvent by liquid CO_2 will result in less harm to ground water.

7. The set in which compounds have different nature is :

(1) $B(OH)_3$ and $Al(OH)_3$
(2) $B(OH)_3$ and H_3PO_3
(3) $Be(OH)_2$ and $Al(OH)_3$
(4) NaOH and $Ca(OH)_2$

Ans. (1)

Sol. $B(OH)_3$ is H_3BO_3 is acidic in nature.

$Al(OH)_3$ is amphoteric in nature.

8. The species given below that does NOT show disproportionation reaction is :

(1) BrO^- (2) BrO_4^- (3) BrO_3^- (4) BrO_2^-

Ans. (2)

Sol. In BrO_4^-, Br is in maximum oxidation state. So it can only reduce.

9. Orlon fibres are made up of :

(1) Polyacrylonitrile (2) Cellulose (3) Polyamide (4) Polyesters

Ans. (1)

Sol. Orlon is a polymer of acrylonitrile also known as PAN

Sol. $$CH_2 = CH - C \equiv N \longrightarrow \left(- CH_2 - \underset{\displaystyle |}{\overset{CN}{CH}} - \right)_n$$

Acrylonitrile PAN or orlon

10. An inorganic compound 'X' on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated H_2SO_4. Also compound 'X' gives precipitated 'Y', when its solution dilute HCl is treated with H_2S gas. The precipitate 'Y' on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound 'X' is :

(1) $Co(NO_3)_2$ (2) $Cu(NO_3)_2$ (3) $Pb(NO_3)_2$ (4) $Pb(NO_2)_2$

Ans. (2)

Sol. Nitrates give brown ring test.

$Cu^{2+} + 4NH_3(aq) \longrightarrow [Cu(NH_3)_4]^{2+}(aq)$

Deep Blue

Cu^2 H_2S ^{H} Cu_2S

Black

11. The correct order of intensity of colors of the compounds is :

(1) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [NiCN)_4]^{2-}$

(2) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

(3) $[NiCl_4]^{2-} > [NiCN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

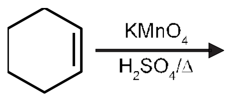
(4) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

Ans. (2)

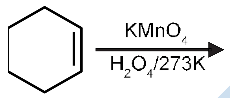
Sol. As all complexes are Ni^{2+} so stronger the ligand greater is splitting and lighter is colour.

Order of strength of ligand $Cl^- < H_2O < CN^-$.

So, order of intensity of colour $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$.

12. 

'A' (Major product)



'B' (Major product)

For above chemical reactions, identify the correct statement from the following.

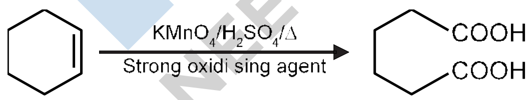
(1) Compound 'A' is diol and compound 'B' is dicarboxylic acid.

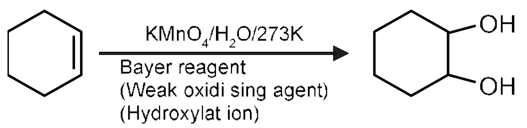
(2) Both compound 'A' and compound 'B' are dicarboxylic acids.

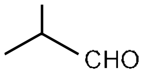
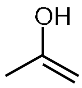
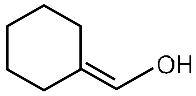
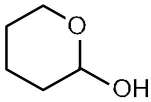
(3) Both compound 'A' and compound 'B' are diols.

(4) Compound 'A' is dicarboxylic acid and compound 'B' is diol.

Ans. (4)

Sol. 



13.  (I)  (II)  (III)  (IV)

Which among the above compound/s does/do not form silver mirror when treated with Tollen's reagent?

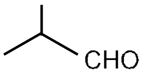
(1) (I), (III) and (IV) only (2) Only (II)

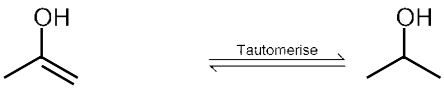
(3) (III) and (IV) only (4) Only (IV)

Ans. (2)

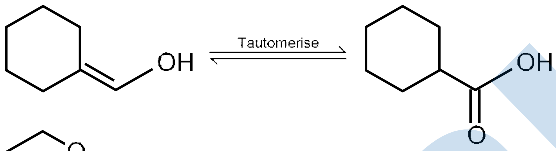
Sol. Aldehyde & hemi acetal group form silver mirror when treated with Tollen's reagent.

Hence compound (I), (III) and (IV) gives this test.

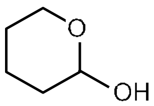
(I)  (Aldehyde)

(II) 

(Ketone does not giv

(III) 

(Aldehyde)

(IV)  (Hemi acetal)

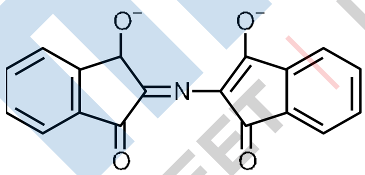
14. The metal that can be purified economically by fractional distillation method is :

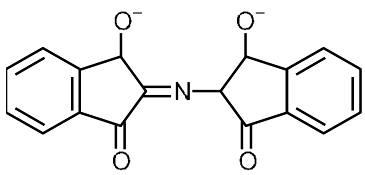
(1) Cu (2) Zn (3) Fe (4) Ni

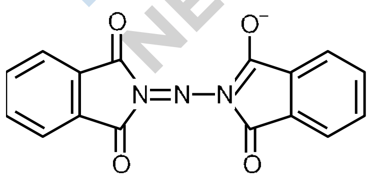
Ans. (2)

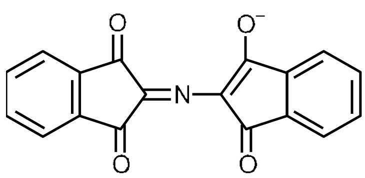
Sol. Fractional distillation process utilizes the boiling point difference between metal and that of impurity. Using this process, crude zinc containing Cd, Fe and Pb as impurities can be refined.

15. The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is :

(1) 

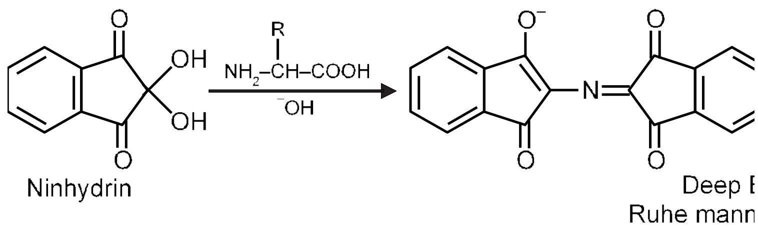
(2) 

(3) 

(4) 

Ans. (4)

Sol.



Ninhydrin is useful for identification of α-amino acid which react with ninhydrin and give deep blue colour.

16. Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric and P_4O_{10} (in 4 : 1 ratio) is :

(1) amphoteric (2) basic (3) neutral (4) acidic

Ans. (4)

Sol. $P_4O_{10} + 4HNO_3 \longrightarrow \underset{'A'}{2N_2O_5} + 4HPO_3$

Nature of oxide 'A' is "acidic".

17. According to the valence bond theory the hybridization of central metal atom is dsp2 for which one of the following compound ?

(1) $Na_2[NiCl_4]$ (2) $[Ni(CO)_4]$ (3) $K_2[Ni(CN)_4]$ (4) $NiCl_2.6H_2O$

Ans. (3)

Sol.

	Complex	Hybridisation
(1)	$Na_2[NiCl_4]$	sp^3
(2)	$[Ni(CO)_4]$	sp^3
(3)	$K_2[Ni(CN)_4]$	dsp^2
(4)	$NiCl_2.6H_2O$	sp^3d^2

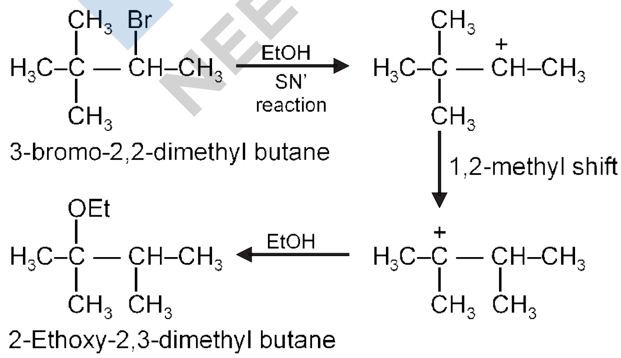
18. In the given reaction 3-Bromo-2,2-dimethyl butane $\xrightarrow{C_2H_5OH}$ 'A' (Major Product), Product A is :

(1) 2-Ethoxy-3,3-dimethyl butane (2) 2-Hydroxy-3,3-dimethyl butane

(3) 2-Ethoxy-2,3-dimethyl butane (4) 1-Ethoxy-3,3-dimethyl butane

Ans. (3)

Sol.



19. The conditions give below are in the context of observing Tyndall effect in colloidal solutions :

(a) The diameter of the colloidal particles is comparable to the wavelength of light used.

(b) The diameter of the colloidal is much smaller than the wavelength of light used.

(c) The diameter of the colloidal particles is much larger than the wavelength of light used.

(d) The refractive indices of the dispersed phase and the dispersion medium are comparable.

(e) The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below :

(1) (C) and (D) only (2) (A) and (D) only (3) (B) and (E) only (4) (A) and (E) only

Ans. (4)

Sol. The conditions give below are in the context of observing Tyndall effect in colloidal solutions :

(a) The diameter of the colloidal particles is comparable to the wavelength of light used.

(e) The refractive index of the dispersed phase and dispersion medium differ greatly in magnitude.

20. A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2. The oxide is pale yellow in colour and paramagnetic. The element (M) is :

(1) Mg (2) K (3) Na (4) Ca

Ans. (2)

Sol. $K + O_2 \text{ (excess)} \longrightarrow KO_2$

Potassium on reaction with excess oxygen give superoxide

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The spin-only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is..........BM.

[At. no. of Co = 27]

Ans. (2)

Sol. $[Co(CN)_6]^{4-} \Rightarrow Co^{2+} \Rightarrow 3d^7\ 4s^0$

So number of unpaired electrons = 1.

$\mu \quad \sqrt{n(n \quad 2)} \quad \sqrt{3}$

μ = 1.73 BM $\approx$ 2 BM.

Note : Answer given by NTA (2) but Zigyan should be (1.73). Since nearest integer not mentioned in the question.

2. An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g mol^{-1}) needed to meet this energy requirement is. ....g.

(Use : $\Delta_C H$(glucose) = –2700 kJ mol^{-1})

Ans. (667)

Sol. $C_6H_{12}O_6 + 6H_2O \longrightarrow 6CO_{2(g)} + 6H_2O$, ΔH = 2700 KJ /mole

Glucose

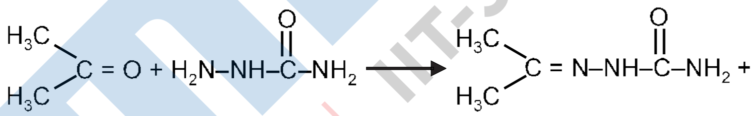
No. of mole of glucose require for production of 10,000 KJ heat is = $\frac{10000}{2700}$ mole.

Total mass of glucose = $\frac{10000}{2700} \quad 180 \quad 666.67$ gram.

3. The number of nitrogen atoms in a semicarbazone molecule of acetone is..............

Ans. (3)

Sol.



semicarbazone

4. The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is ....× 10^{-3} min^{-1}.

[Use : In 10 = 2.303 ; log10 3 = 0.477; Property of logarithm : log x^y = y log x]

Ans. (106)

Sol. $K \quad \frac{1}{t}\ln \frac{a}{a \quad x}$

$\frac{2.303}{1}\log \frac{100}{90}$

$\frac{2.303}{1}\log[\log 10 \quad 2\log 3]$

$= 2.303\,[1 - 2 \times 0.477]$

$= 2.303 \times 0.046 = 0.1059$

$= 105.9 \times 10^{-3} = 106 \times 10^{-3}$

5. 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is..........$\times 10^{21}$. (Nearest integer)

($N_A = 6.022 \times 10^{23}$)

Ans. (226)

Sol. $NaOH + HCl \longrightarrow NaCl + H_2O$

Milimole	125	500	LR is HCl
	0	375	

No. of molecules of HCl remaining unreacted

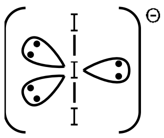
$= 375 \times 10^{-3} \times 6.022 \times 10^{23}$

$= 2.258 \times 10^{23} = 225.8 \times 10^{21} \approx 226 \times 10^{21}$

6. The number of lone pairs of electrons on the central I atom in I_3^- is..........

Ans. (3)

Sol.



Total lone pair on central atom = 3.

7. The Azimuthal quantum number for the valence electrons Ga^+ ion is.........

(Atomic number of Ga = 31)

Ans. (0)

Sol. $Ga = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^1$

$Ga^+ = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2$

Azimuthal Quantum number (ℓ) for valence shell electron is 0.

8. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

In an equilibrium mixture, the partial pressures are

P_{SO_3} 43kPa; P_{O_2} 530Pa and P_{SO_2} 45kPa.

The equilibrium constant K_P = ...$\times 10^{-2}$.

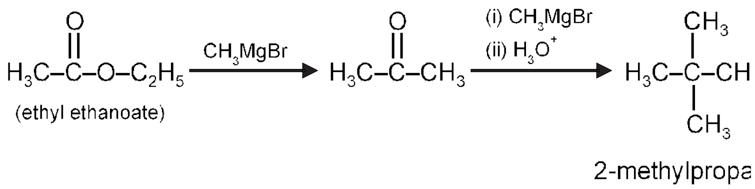
Ans. (172)

Sol. $K_P \quad \frac{(P_{SO_3})^2}{(P_{SO_2})^2(P_{O_2})} \quad \frac{(43)^2}{(45)^2(0.53)} \quad 1.72 \quad 172 \quad 10^{2}$

9. To synthesis 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate……..equivalents of CH_3MgBr reagent will be required.

Ans. (2)

Sol.



10. At 20ºC, the vapour of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20ºC above an equimolar mixture of benzene and methyl benzene is…………..× 10^{-2}.

Ans. (78)

Sol. $P_{Total} \quad P^0_{Benzene}x_{Benzene} \quad P^0_{Toluene}x_{Toluene}$

$(70)\frac{1}{2} \quad (20)\frac{1}{2}$

= 35 + 10

= 45

$P_{Benzene} \quad P_{Total}Y_{Benzene} \quad P^0_{Total}Y_{Benzene}$

$Y_{Benzene} \quad \frac{70 \quad \frac{1}{2}}{45} \quad \frac{35}{45} \quad 0.777 \quad 0.78 \quad 78 \quad 10^{2}$

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The value of the integral $\int_{-1}^{1} \log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is equal to

(1) $\log_e 2 + \frac{\pi}{2} - 1$ (2) $2\log_e 2 + \frac{\pi}{4} - 1$ (3) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$ (4) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

Ans. (1)

Sol. $f(x) = \ell n\left(\sqrt{1-x}+\sqrt{1+x}\right)$ $x \in [-1, 1]$ is an even function

$I = 1\int_0^1 \ell n\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Put x = cos2θ ⇒ dx = – 2sin2θ dθ

∴ $\cos2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$I = 4\int_{\pi/4}^{0} \ell n\,(\sin\theta + \cos\theta)\sqrt{2}\,(-\sin2\theta)\,d\theta$

$= 4\int_0^{\pi/4} \ell n\,(\sin\theta + \cos\theta)\sqrt{2}\,\sin2\theta\,d\theta$

$= 4\left[\ell n(\sin\theta + \cos\theta)\left(-\frac{\cos2\theta}{2}\right)\Big|_0^{\pi/4} - \int_0^{\pi/4}\frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\cdot\left(-\frac{\cos2\theta}{2}\right)d\theta\right] + 4\ell n\sqrt{2}\left(-\frac{\cos2\theta}{2}\right)\Big|_0^{\pi/4}$

$= 4\left(0 + \frac{1}{2}\int_0^{\pi/4}(\cos\theta - \sin\theta)^2 d\theta\right) + 4\ell n\sqrt{2}\left(0 + \frac{1}{2}\right)$

$= 4\left(0 + \frac{1}{2}\int_0^{\pi/4}(1 - \sin2\theta)d\theta\right) + 2\ell n\sqrt{2} = 2\left(\theta + \frac{\cos2\theta}{2}\right)_0^{\pi/4} + \ell n2$

$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) + \ell n2 = \frac{\pi}{2} - 1 + \ell n2$

2. Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to :

(1) $\frac{35}{2}$ (2) $\frac{25}{2}$ (3) $\frac{15}{2}$ (4) 25

Ans. (2)

Sol. Equation of tangent at P(2,2) is T = 0

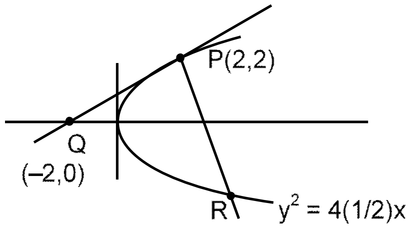
2y = x + 2

So, Q = (–2, 0)

$2at_1 = 2 \Rightarrow t_1 = 2$

$t_2 \quad t_1 \quad \frac{2}{t_1} \quad 2 \quad \frac{2}{2} \quad 3$

$R \quad \frac{1}{3}(\ 3)^2, 2 \quad \frac{1}{2} \quad (\ 3) \qquad \frac{9}{2}, \ 3$



Area of $\Delta PQR \quad \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ 2 & 0 & 1 \\ \frac{9}{2} & 3 & 1 \end{vmatrix}$

$\frac{1}{2}[2(0 \quad 3) \quad 2(\ 2 \quad 9/2) \quad 1(6 \quad 0)] \quad \frac{1}{2}[6 \quad 4 \quad 9 \quad 6] \quad \frac{25}{2}$ sq. unit

3. The coefficient of x^{256} in the expansion of $(1 – x)^{101} (x^2 + x + 1)^{100}$ is :

(1) $– {}^{100}C_{15}$ (2) $–{}^{100}C_{16}$ (3) ${}^{100}C_{16}$ (4) ${}^{100}C_{15}$

Ans. (4)

Sol. $\Rightarrow (1 – x)^{101}(x^2 + x + 1)^{100}$

$\Rightarrow (1 – x)^{100}(x^2 + x + 1)^{100} (1 – x)$

$\Rightarrow (1 – x^3)^{100} (1 – x)$

$\Rightarrow (1 \quad x)({}^{100}C_0 \quad {}^{100}C_{1x^3} \quad {}^{100}C_{2x^6} \quad .... \quad {}^{100}C_{84x^{252}} \quad {}^{100}C_{85x^{255}} \quad {}^{100}C_{86x^{256}} \quad ......)$

$\Rightarrow \ {}^{100}C_{85x^{256}}$

so, the coefficient of x^{256} is ${}^{100}C_{85}$

4. Let a be a positive real number such that $\int_0^a e^{x \ [x]}dx \quad 10e \quad 9$ where [x] is the greatest integer less than or equal to x. Then a is equal to :

(1) $10 + \log_e(1 + e)$ (2) $10 – \log_e(1 + e)$ (3) $10 + \log_e 2$ (4) $10 + \log_e 3$

Ans. (3)

Sol. Let a = 10 + K, $0 \le K < 1$

$\int_0^a e^{\{x\}}dx \quad 10e \quad 9$

$\int_0^{10} e^{\{x\}}dx \quad \int_{10}^{10\ K} e^{[x]}dx \quad 10e \quad 10 \quad 1 \qquad \int_{10}^{10\ K} e^{\{x\}}dx \quad 1 \qquad \int_0^{K} e^{\{x\}}dx \quad 1$

$e^K – 1 = 1$

K = ln2

So, a = 10 + ln2

5. Let y = y(x) be the solution of the differential equation

$x\tan\left(\frac{y}{x}\right)dx = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,\ -1 \le x \le 1,\ y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves

x = 0, $x = \frac{1}{\sqrt{2}}$ and y = y(x) in the upper half plane is :

(1) $\frac{1}{6}(\pi - 1)$ (2) $\frac{1}{12}(\pi - 3)$ (3) $\frac{1}{8}(\pi - 1)$ (4) $\frac{1}{4}(\pi - 2)$

Ans. (3)

Sol. $x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx$

$\Rightarrow \tan\left(\frac{y}{x}\right)dy = \left(\frac{y}{x}\tan\left(\frac{y}{x}\right) - 1\right)dx$

let $\frac{y}{x} = t$

$\frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = \frac{t\tan t - 1}{\tan t}$

$t + \frac{xdt}{dx} = t - \cot t$

$\int \tan t\,dt = -\int \frac{dx}{x} \Rightarrow \log|\sec t| = -\ln|x| + \ln c$

$\ln\left|\sec\left(\frac{y}{x}\right)\right| = \ln\left|\frac{c}{x}\right|$

When x > 0, y > 0 then

$\sec\left(\frac{y}{x}\right) = \frac{c}{x}$

$y\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sec\left(\frac{\pi}{3}\right) = \frac{c}{\frac{1}{2}}$

c = 1

$\sec\left(\frac{y}{x}\right) = \frac{1}{x}$

$\frac{y}{x} \quad \sec^{-1}\frac{1}{x}$

$y \quad x\sec^{-1}\frac{1}{x}$

Area $\int_0^{\frac{1}{\sqrt{2}}} x\sec^{-1}\frac{1}{x}\,dx \quad \int_0^{\frac{1}{\sqrt{2}}} x\cos^{-1}x\,dx$

$\frac{x^2}{2}\cos^{-1}x\Big|_0^{\frac{1}{\sqrt{2}}} \quad \int_0^{\frac{1}{\sqrt{2}}} \frac{x^2}{2\sqrt{1 \quad x^2}}$

$\frac{1}{4}.\frac{}{4} \quad 0 \quad \frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{1 \quad (1 \quad x^2)}{\sqrt{1 \quad x^2}}dx$

$\frac{}{16} \quad \frac{1}{2} \quad \sin^{-1}x\Big|_0^{\frac{1}{\sqrt{2}}} \quad \int_0^{\frac{1}{\sqrt{2}}} \sqrt{1 \quad x^2}\,dx$

$\frac{}{16} \quad \frac{1}{2} \quad \frac{}{4} \quad \frac{1}{2}x\sqrt{1 \quad x^2} \quad \frac{1}{2}\sin^{-1}x\Big|_0^{\frac{1}{\sqrt{2}}} \quad \frac{}{16} \quad \frac{1}{2} \quad \frac{}{4} \quad \frac{}{8} \quad \frac{1}{8}$

6. Let a be a real number such that the function $f(x) = ax^2 + 6x - 15$, $x \in R$ is increasing in $\left(, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4},\right)$. The function $g(x) = ax^2 - 6x - 15$, $x \in R$ has a :

(1) local minimum at $x \quad \frac{3}{4}$

(2) local maximum at $x \quad \frac{3}{4}$

(3) local minimum at $x \quad \frac{3}{4}$

(4) local maximum at $x \quad \frac{3}{4}$

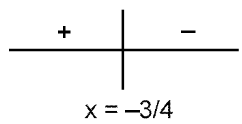
Ans. (2)

Sol. $f(x) = ax^2 + 6x - 15$

f$'(x) = 2ax + 6$

For maxima & minima $f'(x) = 0 \Rightarrow x = -3/a$

According to the question, $\frac{3}{a} \quad \frac{3}{4} \quad a \quad 4$



Then $g(x) = -4x^2 - 6x + 15$

$g'(x) = -8x - 6 = 0$

$x \quad \frac{3}{4}$

$x \quad \frac{3}{4}$ is a point of local maxima

7. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is :

(1) 1 (2) 4 (3) 0 (4) 2

Ans. (3)

Sol. $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$

as $x^2 + x \geq 0 \Rightarrow x^2 + x + 1 \geq 1$

But $x^2 + x + 1 \leq 1$

So, $x^2 + x = 0$

$x = 0, -1$

$x = 0, -1$ does not satisfies the original equation

$\therefore$ no solution

8. Let a function $f : R \to R$ be defined as $f(x) = \begin{cases} \sin x - e^x & ; \text{if } x \leq 0 \\ a + [-x] & ; \text{if } 0 < x < 1 \\ 2x - b & ; \text{if } x \geq 1 \end{cases}$ where [x] is the greatest integer less than or equal to x. If f is continuous on R, then (a + b) is equal to :

(1) 4 (2) 5 (3) 3 4) 2

Ans. (3)

Sol. Since f(x) is continuous at x = 0

So $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = f(0)$

$-1 = a - 1 = -1 \Rightarrow a = 0$

Since f(x) is continuous at x = 1

So $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)$

$a - 1 = 2 - b = 2 - b$

$\Rightarrow a = 0$, so $0 - 1 = 2 - b$

$\Rightarrow -3 = -b$

$\Rightarrow b = 3$

So the value of a + b = 3

9. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are :

(1) 3, 18 (2) 10, 11 (3) 1, 20 (4) 8, 13

Ans. (2)

Sol. Let two number x and y according to equation

$18 + x + y = 39$

$x + y = 21$ ....(1)

$10.25 \quad \frac{x_i^2}{n} \quad (\bar{x})^2$

$10.25 \quad \frac{x^2 \quad y^2 \quad 4 \quad 16 \quad 25 \quad 49}{6} \quad (6.5)^2$

$10.25 \quad \frac{x^2 \quad y^2 \quad 94}{6} \quad (6.5)^2$

$\Rightarrow x^2 + y^2 = 221$ ....(2)

Solving (1) & (2)

So, x = 10 or y = 11

10. Let A = $[a_{ij}]$ be a 3 × 3 matrix, where $a_{ij} \quad \begin{matrix} 1 & , \text{ if } i \quad j \\ x & , \text{ if} |i \quad j| \quad 1 \\ 2x \quad 1 & , \text{otherwise} \end{matrix}$. Let a function f : R → R be defined as f (x) = det(A). Then the sum of maximum and minimum values of f or R is equal to :

(1) $\frac{20}{27}$ (2) $\frac{20}{27}$ (3) $\frac{88}{27}$ (4) $\frac{88}{27}$

Ans. (3)

Sol. $|A| \begin{vmatrix} 1 & x & 2x \quad 1 \\ x & 1 & x \\ 2x \quad 1 & x & 1 \end{vmatrix} \quad 1 \quad x^2(2x \quad 1) \quad x^2(2x \quad 1) \quad (2x \quad 1)^2 x^2 \quad x^2$

$\Rightarrow$ f(x) = $4x^3 - 4x^2 - 4x$

$\Rightarrow$ f(x) = $12x^2 - 8x - 4$

+ | − | +

$\frac{-1}{3}$ 1

= $4(3x^2 - 2x - 1)$

= $4(x - 1)(3x + 1)$

$\Rightarrow$ f(x) is maximum at $x \quad \frac{1}{3}$ and minimum at x = 1

$\therefore$ maximum value $\frac{20}{27}$ and minimum value = –4

$\therefore$ sum $\frac{20}{27} \quad 4 \quad \frac{88}{27}$

11. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(1) $P \Rightarrow \sim q$ (2) $\sim q \Rightarrow P$ (3) $q \Rightarrow p$ (4) $p \Rightarrow q$

Ans. (4)

Sol. $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$\Rightarrow (\sim p \vee q) \vee (q \vee \sim p)$

$\Rightarrow \sim p \vee q$

$= p \Rightarrow q$

12. Let [x] denote the greatest integer $\leq$ x, where x $\in$ R. If the domain of the real valued function

$f(x) \quad \sqrt{\dfrac{|[x]| \quad 2}{|[x]| \quad 3}}$ is $(-\infty, a) \cup [b, c] \cup [4, \infty)$, a < b < c, then the value of a + b + c is :

(1) 1 (2) –2 (3) 8 (4) –3

Ans. (2)

Sol. $\dfrac{|[x]| \quad 2}{|[x]| \quad 3} \quad 0 \quad |[x]| \quad 3 \quad 0$

Let t $\in$ |[x]|, t > 0

$\Rightarrow \quad \dfrac{t \quad 2}{t \quad 3} \quad 0$

$\Rightarrow \quad t \in (-\infty, 2) \cup (3, \infty) \cap t > 0$

$\Rightarrow \quad |[x]| \in [0, 2] \cup (3, \infty)$

$\Rightarrow \quad [x] \in (-\infty, 3) \cup [-2,2] \cup (3, \infty)$

$\Rightarrow \quad [x] \in (-\infty, -3) \cup [-2,3] \cup (4, \infty)$

So a = –3, b = –2, c = 3

So a + b + c = –2

13. Let y = y(x) be the solution of the differential equation $e^x\sqrt{1 \quad y^2}dx \quad \dfrac{y}{x}\,dy \quad 0, y(1) \quad 1$. Then the value of $(y(3))^2$ is equal to :

(1) $1 + 4e^6$ (2) $1 + 4e^3$ (3) $1 - 4e^3$ (4) $1 - 4e^6$

Ans. (4)

Sol. $e^x\sqrt{1 \quad y^2}dx \quad \dfrac{y}{x}dy$

$xe^x dx \quad \dfrac{y}{\sqrt{1 \quad y^2}}dy$

Put $1 - y^2 = t^2$

$-2y\,dy = 2t\,dt$

$xe^x \quad e^x \quad \dfrac{tdt}{t}$

$xe^x \quad e^x \quad \sqrt{1 \quad y^2} \quad c$

Given y(1) = –1

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$xe^x \quad e^x \quad \sqrt{1 \quad y^2}$

Put $x = 3$

$(3 \quad 1)e^{3} \quad \sqrt{1 \quad y^{2}}$

$(y(3))^{2} = 1 - 4e^{6}$

14. If α and β are the distinct roots of the equation $x^{2} + (3)^{1/4} x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1)$ $\beta^{96}(\beta^{12} - 1)$ is equal to :

(1) 56×3^{24} (2) 52×3^{24} (3) 56×3^{25} (4) 28×3^{25}

Ans. (2)

Sol. $x^{2} \quad \sqrt{3} \quad 3^{\frac{1}{4}} x$

$x^{4} \quad 2\sqrt{3}x^{2} \quad 3 \quad \sqrt{3}x^{2}$

$x^{4} \quad \sqrt{3}x^{2} \quad 3 \quad 0$

$\Rightarrow x^{8} + 6x^{4} + 9 = 3x^{4}$

$\Rightarrow x^{8} + 3x^{4} + 9 = 0$

$\Rightarrow \alpha^{8} - 9\alpha^{4} - 3\alpha^{8} = -9\alpha^{4} - 3(-9-3\alpha^{4}) = 27$

Similarly $\beta^{12} - 27$

$\Rightarrow \alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1) = (27)^{8} . 26 + (27)^{8} . 26 \; 52. (27)^{8} = 52.3^{24}$

15. If in a triangle ABC, AB = 5 units, $B \quad \cos^{1} \frac{3}{5}$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

(1) $4 \quad 2\sqrt{3}$ (2) $8 \quad 2\sqrt{2}$ (3) $6 \quad 8\sqrt{3}$ (4) $10 \quad 6\sqrt{2}$

Ans. (3)

Sol. $\cos B \quad \frac{3}{5} \quad \sin B \quad \frac{4}{5}, R \quad 5$

$\frac{b}{2R} \quad \frac{4}{5} \quad b \quad 8, c \quad 5$

$\cos B \quad \frac{a^{2} \quad c^{2} \quad b^{2}}{2ac} \quad \frac{3}{5} \quad \frac{a^{2} \quad 25 \quad 64}{2a(5)} \quad \frac{3}{5}$

$a^{2} - 39 = 6a \Rightarrow a^{2} - 6a - 39 = 0$

$a \quad \frac{6 \quad 8\sqrt{3}}{2} \quad a \quad 3 \quad 4\sqrt{3}$

$\frac{abc}{4R} \quad \frac{3 \quad 4\sqrt{3}}{4(5)} \quad 6 \quad 8\sqrt{3}$

16. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

(1) $\frac{1}{11}$ (2) $\frac{1}{66}$ (3) $\frac{2}{11}$ (4) $\frac{1}{9}$

Ans. (1)

Sol. EXAMINATION

$E \rightarrow 1$ $\quad n(S)\ \frac{11!}{2!2!2!}$

$X \rightarrow 1$ $\quad n(E)\ \frac{10!}{2!2!2!}$

$A \rightarrow 2$ $\quad P(E)\ \frac{n(E)}{n(s)}\ \frac{1}{11}$

$M \rightarrow 1$

$O \rightarrow 1$

$T \rightarrow 1$

$N \rightarrow 2$

$I \rightarrow 2$

17. The probability of selecting integers $a \in [-5,30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$ for all $x \in R$, is :

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{4}$

Ans. (2)

Sol. $x^2 + 2(a+4)x - (5a - 64) > 0$

$D < 0$

$\therefore 4(a+4)^2 + 4(5a - 64) < 0$

$\Rightarrow (a+4)^2 + (5a - 64) < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$a\ \frac{13\ \sqrt{168\ \ 192}}{2}\ \ 16, 3$

So, $a \in (-16, 3)$

So, $a = -5, -4, -3, -2, -1, 0, 1, 2$

$\therefore$ Required Probability $= \frac{8}{36}\ \frac{2}{9}$

18. Let $\vec{a}\ \ 2\hat{i}\ \ \hat{j}\ \ 2\hat{k}$ and $\vec{b}\ \ \hat{i}\ \ \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\ \vec{c}\ \ |\vec{c}|, |\vec{c}\ \ \vec{a}|\ \ 2\sqrt{2}$ and the angle between $(\vec{a}\ \ \vec{b})$ and $\vec{c}$ is $\frac{}{6}$, then the value of $\left|(\vec{a}\ \ \vec{b})\ \ \vec{c}\right|$ is :

(1) $\frac{2}{3}$ (2) 3 (3) $\frac{3}{2}$ (4) 4

Ans. (3)

Sol. $\vec{a} \quad \vec{b} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$

$\hat{i}(2) \quad \hat{j}(2) \quad \hat{k}(1) \qquad\qquad 2\hat{i} \quad 2\hat{j} \quad \hat{k}$

$|\vec{a} \quad \vec{b}| \quad 3$

$|\vec{c} \quad \vec{a}| \quad 8$

$|\vec{c}|^2 \quad |\vec{a}|^2 \quad 2\vec{a} \quad \vec{c} \quad 8$

$|\vec{c}|^2 \quad 9 \quad 2|\vec{c}| \quad 8$

$|\vec{c}|^2 \quad 2|\vec{c}| \quad 1 \quad 8$

$|\vec{c}| \quad 1$

$|(\vec{a} \quad \vec{b}) \quad \vec{c}| \; |\vec{a} \quad \vec{b}||\vec{c}| \sin\frac{}{6}$

$3 \quad 1 \quad \frac{1}{2} \quad \frac{3}{2}$

19. Let A $\begin{matrix} 2 & 3 \\ a & 0 \end{matrix}$,a R be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 45 (2) 24 (3) 18 (4) 36

Ans. (4)

Sol. $\because A \quad \frac{1}{2}(A \quad A^T) \quad \frac{1}{2}(A \quad A^T)$

Where $A + A^T$ is symmetric and $(A - A^T)$ is skew symmetric

$P \quad \frac{1}{2}(A \quad A^T)$ and $A \quad \frac{1}{2}(A \quad A^T)$

$Q \quad \frac{1}{2} \begin{matrix} 0 & 3 \quad a \\ a \quad 3 & 0 \end{matrix}$

$\det(Q) \quad \frac{1}{4}(a \quad 3)^2 \quad 9$

$\Rightarrow (a - 3)^2 = 36$

$\Rightarrow a = 9$ or -3

Now $P \quad \frac{1}{2} \begin{matrix} 4 & 3 \quad a \\ a \quad 3 & 0 \end{matrix}$

$\det(P) \quad \frac{1}{4}(a \quad 3)^2 \qquad 36$ or 0

$\Rightarrow$ sum of all possible values of det (P) = 36

20. If z and ω are two complex numbers such that $|z \quad | \quad 1, \arg(z) \quad \arg(\quad) \quad \frac{3}{2}$, then $\arg \frac{1 \quad 2\bar{a}}{1 \quad 3\bar{z}}$, is :

(Here arg (z) denotes the principal argument of complex number z)

(1) $\frac{}{4}$ (2) $\frac{3}{4}$ (3) $\frac{3}{4}$ (4) $\frac{}{4}$

Ans. (3)

Sol. $|z \quad | \quad 1$ and $\arg(z) \quad \arg(\quad) \quad \frac{3}{2}$

$|z|| \quad | \quad 1$ and $\arg(z) \quad \arg(\quad) \quad \frac{3}{2}$

Let $w = re^{i\theta}$

$$z \quad \frac{1}{r}e^{i \quad \frac{3}{2}}$$

$$\frac{1 \quad 2\bar{z}}{1 \quad 3\bar{z}} \quad \frac{1 \quad 2re^{i} \quad \frac{1}{r}e^{i \quad \frac{3}{2}}}{1 \quad 3re^{ei} \quad \frac{1}{r}e^{i \quad \frac{3}{2}}}$$

$$\frac{1 \quad 2e^{i3 \quad /2}}{1 \quad 3e^{\quad i3 \quad /2}} \quad \frac{1 \quad 2i}{1 \quad 3i}$$

$$\arg \frac{1 \quad 2\bar{z}}{1 \quad 3\bar{z}} \quad \arg \frac{1 \quad 2i}{1 \quad 3i} \quad \tan^{1}(\quad 2) \quad \tan^{1}(3) \quad \frac{3}{4}$$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = 7A^{20} - 20A^{7} + 2I$, where I is an identity matrix or order 3 × 3. If $B = [b_{ij}]$,

then b_{13} is equal to :

Ans. (910)

Sol. Let A = I + C

Where $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$C^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$C^3 = 0$

So $A^2 = (I + C)^2 = I + 2C + C^2$

$A^3 = A^2 . A = I + 3C + 3C^2$

$A^4 = I + 4C + 6C^2$

$A^5 = I + 5C + 10C^2$

So, $A^n = I + nC + \frac{n(n-1)}{2}C^2$

$A^{20} = I + 20C + 190C^2$

$A^7 = I + 7C + 21C^2$

$B = 7A^{20} - 20A^7 + 2I$

$B = 11I + 910C^2 = \begin{bmatrix} 11 & 0 & 910 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$\therefore$ $b_{13} = 910$

2. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to :

Ans. (4)

Sol. $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 3$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$

Now $\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}||\vec{a} + \vec{b} + \vec{c}|\cos\theta$

$$\cos \quad \frac{1}{\sqrt{3}} \quad \cos 2 \quad 2\cos^2 \quad 1$$

$$\cos 2 \quad \frac{1}{3} \quad \cos^2 2 \quad \frac{1}{9} \quad 36\cos^2 2 \quad 4$$

3. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. Then number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is :

Ans. (777)

Sol. **Case-I :** Team consist 5 Batsman , 5 Bowlers

and 1 wicket keeper then number of ways.

$= {}^6C_5 \times {}^7C_5 \times {}^2C_1 = 6 \times 21 \times 2 = 252$

Case-II : 4 Batsmen, 6 bowlers and 1 wicket keeper

$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 = 15 \times 7 \times 2 = 210$

Case-III : 4 Batsmen, 5 Bowlers and 2 wicket keepers

${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 15 \times 21 \times 1 = 315$

Total 252 + 210 + 315 = 777

4. If the shortest distance between the lines $\vec{r}_1 \quad \hat{i} \quad 2\hat{j} \quad 2\hat{k} \quad (\hat{i} \quad 2\hat{j} \quad 2\hat{k}), \quad R, \quad 0$ and

$\vec{r}_2 \quad 4\hat{i} \quad \hat{k} \quad \mu(3\hat{i} \quad 2\hat{j} \quad 2\hat{k}), \mu \quad R$ is 9, then α is equal to :

Ans. (6)

Sol. Shortest distance $\left|\frac{(a_2 \quad a_1)\ (b_1 \quad b_2)}{|b_1 \quad b_2|}\right|$

$$9 \quad \left|\frac{((\quad 4)\hat{i} \quad 2\hat{j} \quad 3\hat{k})\ (8\hat{i} \quad 8\hat{j} \quad 4\hat{k})}{\sqrt{64 \quad 64 \quad 16}}\right| \quad \left|\frac{8(\quad 4) \quad 16 \quad 12}{12}\right| \quad 9$$

$\therefore \quad \alpha = 6$

5. The number of rational terms in the binomial expansion of $4^{\frac{1}{4}} \quad 5^{\frac{1}{6}} \quad ^{120}$ is :

Ans. (21)

Sol. General term of $2^{\frac{1}{2}} \quad 5^{\frac{1}{6}} \quad ^{120}$ is

given by $T T_{t\ 1} \quad {}^{120}C_r \quad 2^{\frac{1}{4}} \quad ^{120\ r} \quad 5^{\frac{1}{6}} \quad ^{r}$

For integral term, r should be a multiple of 6

i.e $\quad r \in \{0,6,12,18, \ldots\ldots., 120\}$

$\therefore$ 21 integral terms are there in the expansion $2^{\frac{1}{2}} \quad 5^{\frac{1}{6}} \quad ^{20}$

6. Let P be a plane passing through the points (1, 0, 1),(1, –2, 1) and (0, 1, –2). Let a vector $\vec{a}\quad \hat{i}\quad \hat{j}\quad \hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i}\quad 2\hat{j}\quad 3\hat{k})$ and $\vec{a}\ (\hat{i}\quad \hat{j}\quad 2\hat{k})\quad 2$, then $(\alpha - \beta + \gamma)^2$ equals :

Ans. (81)

Sol. Equation of plane P is

$$\begin{vmatrix} x \quad 1 & y \quad 0 & z \quad 1 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{vmatrix} \quad 0$$

$\Rightarrow -3(x + 1) + (z - 1) = 0$

$\Rightarrow 3x - z - 2 = 0$

Normal vector to the plane P is $\vec{n}\quad 3\hat{i}\quad \hat{k}$.

Now $\vec{a}\quad \hat{i}\quad \hat{j}\quad \hat{k}$ is perpendicular to $\vec{n}$

$\vec{a}\ \vec{n}\quad 0$

$\Rightarrow 3\alpha - \gamma = 0$ ....(1)

Also $\vec{a}$ is perpendicular to $\vec{b}\quad \hat{i}\quad 2\hat{j}\quad 3\hat{k}$

$\vec{a}\ \vec{b}\quad 0$

$\Rightarrow \alpha + 2\beta + 2\gamma = 0$ ....(2)

and $\vec{a}\ (\hat{i}\quad \hat{j}\quad 2\hat{k})\quad 2 \qquad 2 \quad 2$ ....(3)

by solving (1), (2) and (3)

$\alpha = 1, \beta = -5, \gamma = 3 \Rightarrow (\alpha - \beta + \gamma)^2 = 81$

7. Let a, b, c, d be in arithmetic progression with common difference λ. If $\begin{vmatrix} x \quad a \quad c & x \quad b & x \quad a \\ x \quad 1 & x \quad c & x \quad b \\ x \quad b \quad d & x \quad d & x \quad c \end{vmatrix} \quad 2$, then value of λ^2 is equal to :

Ans. (1)

Sol. $\begin{vmatrix} x \quad a \quad c & x \quad b & x \quad a \\ x \quad 1 & x \quad c & x \quad b \\ x \quad b \quad d & x \quad d & x \quad c \end{vmatrix} \quad 2$

$$\begin{vmatrix} x \quad 2 & x \quad b & x \quad a \\ x \quad 1 & x \quad c & x \quad b \\ x \quad 2 & x \quad d & x \quad c \end{vmatrix} \quad 2 \ (\because c \quad a \quad 2 \quad d \quad b)$$

$R_2 \to R_2 - R_1 \ \ \& \ R_3 \to R_3 - R_2$

$$\begin{vmatrix} x \quad 2 & x \quad b & x \quad a \\ 1 \quad 2 & & \\ 2 \quad 1 & & \end{vmatrix} \quad 2$$

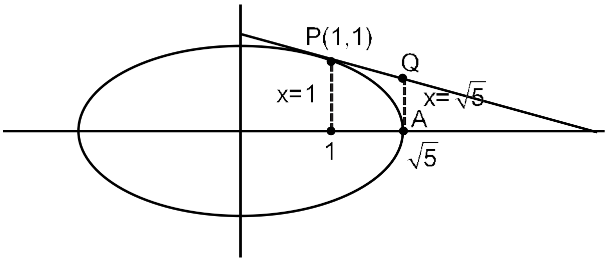
$R_3 \rightarrow R_3 - R_2$

$$\begin{vmatrix} x & 2 & x \quad b \quad x \quad a \\ 2 & 1 & \\ 2 & 0 & 0 \end{vmatrix} \quad 2 \quad 2\,(x \quad b \quad x \quad a) \quad 2 \quad {}^{2} \quad 1$$

8. Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and x $\sqrt{5}$ is $\sqrt{5}$ $\cos^{1}\frac{1}{\sqrt{5}}$, then |α + β + γ| is equal to :

Ans. (1.25)

Sol.



Equation of tangent at P (1,1) is x + 4y = 5

Now area bounded by the required region

$$\int_{1}^{\sqrt{5}} \frac{5 \quad x}{4} \quad \sqrt{\frac{5 \quad x^2}{4}}\, dx$$

$$\frac{5}{4}x \quad \frac{x^2}{8}\Bigg|_{1}^{\sqrt{5}} \quad \frac{1}{2}\;\frac{1}{2} \quad \sqrt{5 \quad x^2} \quad \frac{5}{2}\sin^{1}\frac{x}{\sqrt{5}}\Bigg|_{1}^{\sqrt{5}}$$

$$\frac{5\sqrt{5}}{4} \quad \frac{5}{8} \qquad \frac{5}{4} \quad \frac{1}{8} \quad \frac{1}{2} \quad 0 \quad \frac{5}{4} \qquad 1 \quad \frac{5}{2}\sin^{1}\frac{1}{\sqrt{5}}$$

$$\frac{5}{4} \quad \sqrt{5} \quad 1 \quad \frac{1}{2} \quad \frac{5}{8} \quad \frac{1}{2} \quad \frac{5}{4}\sin^{1}\frac{1}{\sqrt{5}}$$

$$\frac{5\sqrt{5}}{4} \quad \frac{5}{4} \quad \frac{5}{4}\cos^{1}\frac{1}{\sqrt{5}} \qquad \frac{5}{4}, \qquad \frac{5}{4} \text{ and } \qquad \frac{5}{4}$$

$$| \qquad | \quad \frac{5}{4}$$

9. Let y = mx + c, m > 0 be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}$ (m + c) is equal to :

Ans. (34)

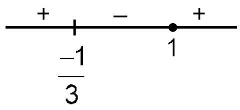
Sol. $|A| \begin{vmatrix} 1 & x \quad 2x & 1 \\ x & 1 & x \\ 2x \quad 1 & x & 1 \end{vmatrix} \quad 1 \quad x^2(2x \quad 1) \quad x^2(2x \quad 1) \quad (2x \quad 1)^2 \quad x^2 \quad x^2$

$\Rightarrow f(x) = 4x^3 - 4x^2 - 4x$

$\Rightarrow f'(x) = 12x^2 - 8x - 4$

$= 4(3x^2 - 2x - 1)$

$= 4(x - 1)(3x + 1)$



$\Rightarrow f(x)$ is maximum at $x = -\frac{1}{3}$ and minimum at $x = 1$

$\therefore$ maximum value $= \frac{20}{27}$ and minimum value $= -4$

$\therefore$ sum $= \frac{20}{27} - 4 = -\frac{88}{27}$

10. If the value $\lim_{x \to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\frac{x+2}{x^2}}$ is equal to e^a, then a is equal to

Ans. (3)

Sol. $\lim_{x \to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\frac{x+2}{x^2}} = e^{\lim_{x \to 0}\frac{(1 - \cos x\sqrt{\cos 2x})(x+2)}{x^2}}$

$= e^{\lim_{x \to 0}\frac{\left(1 - \cos^2 x \cos 2x\right)(x+2)}{x^2(1 + \cos x\sqrt{\cos 2x})}} = e^{\lim_{x \to 0}\frac{(1 - (1 - \sin^2 x)(1 - 2\sin^2 x))(x+2)}{x^2(1 + \cos x\sqrt{\cos 2x})}}$

$= e^{\lim_{x \to 0}\frac{\left(3\sin^2 x - 2\sin^4 x\right)(x+2)}{x^2(1 + \cos x\sqrt{\cos 2x})}}$

$= e^{\lim_{x \to 0}\frac{\sin^2 x}{x^2} \cdot \frac{\left(3 - 2\sin^2 x\right)(x+2)}{(1 + \cos x\sqrt{\cos 2x})}}$

$= e^{\frac{3 \times 2}{1+1}}$

$= e^3$

$\therefore a = 3$